Real Options, Energy Finance, and the Current World Situation

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Go20 Conference

Western Science

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Quantitative Energy Finance

Price Model

- Deterministic:
  world oil demand → world oil price
- Stochastic:
  E.g., $dS = \mu S dt + \sigma S dW$

Value Formula

- How to exploit impact of oil production by a country/company on oil price?
- Value a call option with $V(S,T) = \max(S_T - K, 0)$

Commodities

Valuable for What They Bring

- Oil: transportation & plastic
- Gas: heat, electricity generation
- Electricity: motive power, air conditioning
- All are difficult to store and transport

Spreads

- Calendar
- Location
- Spark
- Crack
- Crush
- $V(R,S) = \max(aR-bS-K,0)$

Real Options

Spread	Infrastructure
Gas Calendar Spread	Salt Dome Storage
Oil Location Spread	Oil Tanker; Pipeline
Spark Spread	Natural Gas Power Plant
Crack Spread	Oil Refinery
Crush Spread	Corn Ethanol

Pricing Real Options

Facility = Sum of Spread Options

- PDE and exercise decision
- Physics of problem impacts exercise decisions
  - Have to fill a facility before you can empty it;
  - Can't switch things on and off too frequently

Results

- Stochastic Optimal control PDE
- Lots of nice numerical analysis
- Gives value functions; control surfaces

Assumptions and Results

Assumptions

- Owners of assets have control over them
- They use their power to extract profits
- The more market uncertainty the more the profit

Results

- Gives value to flexible assets
- Assets are used to stabilize the market
- Store gas in cheap season; use in expensive season
- Move oil from cheap market to expensive market

Risk vs. Uncertainty

Regular Uncertainty

- Corn ethanol market has lots of kinds of uncertainty
- Fluctuating subsidies
- Blend mandate

But …

- Subsidies change behaviour
- Behavioural changes blunt subsidy impact

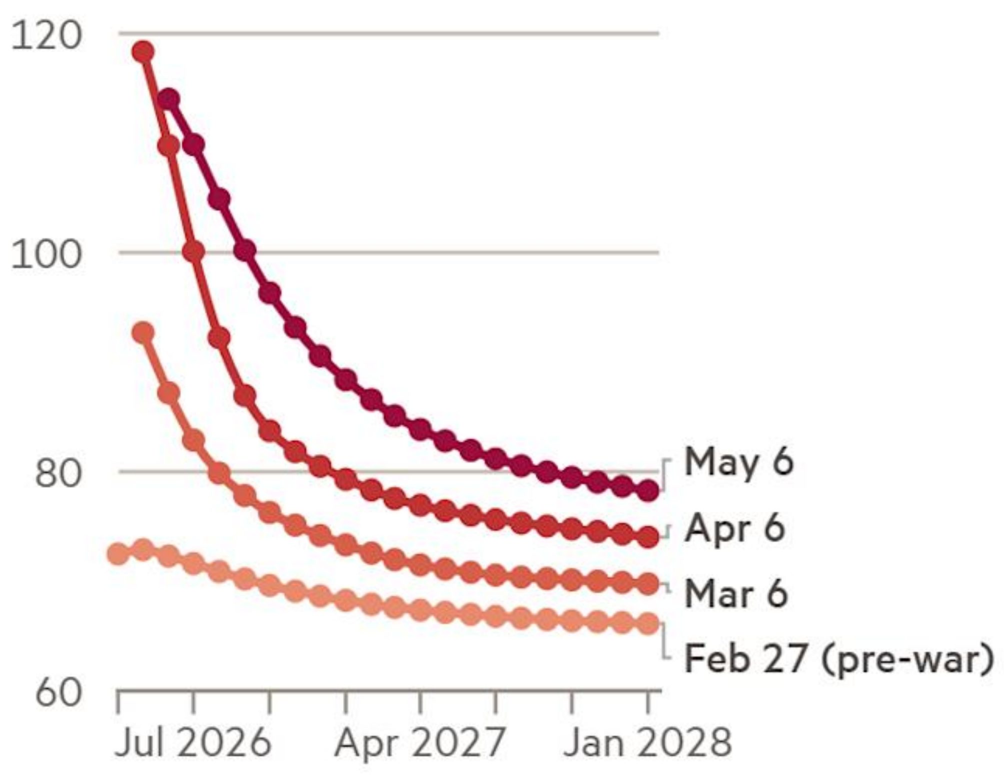
  → homeostasis

What About the Present?

Do Financial Markets even make sense right now?

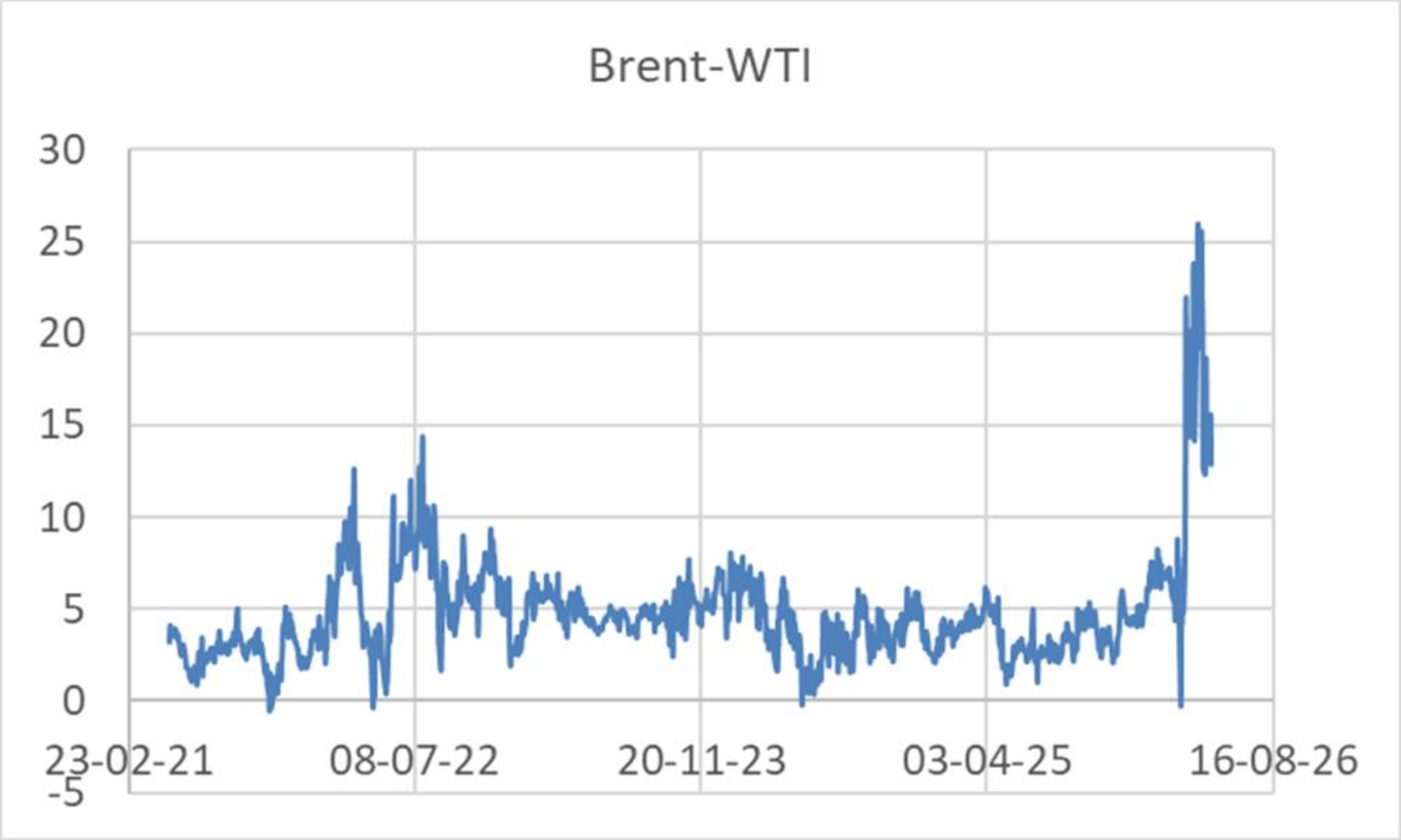
Since the war began, near-dated oil has gained far more than long-dated oil

Brent crude oil futures ($ per barrel), by date



©*FT* Source: LSEG

Location Spread



(data from FRED)

Energy Storage

GULF NEWS DUBAI 27°C GOLD/FOREX PR

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Business / Energy

Iran scrambles to store more oil as Kharg Island nears capacity under blockade pressure

Floating storage buys time as blockade pushes Iran's main oil terminal to brink

Last updated: April 24, 2026 | 09:40

Jay Hilotin, *Senior Assistant Editor*

REUTERS Crude oil seen stored on tankers in 2015 as contango widens

COMMODITIES | Tue Dec 23, 2014 | 3:00pm EST

Crude oil seen stored on tankers in 2015 as contango widens

Oil tankers are pictured docked at the Guanabara Bay in the state of Rio de Janeiro, November 19, 2014. REUTERS/Pilar Olivares

Jet Fuel Price: Crack Spread

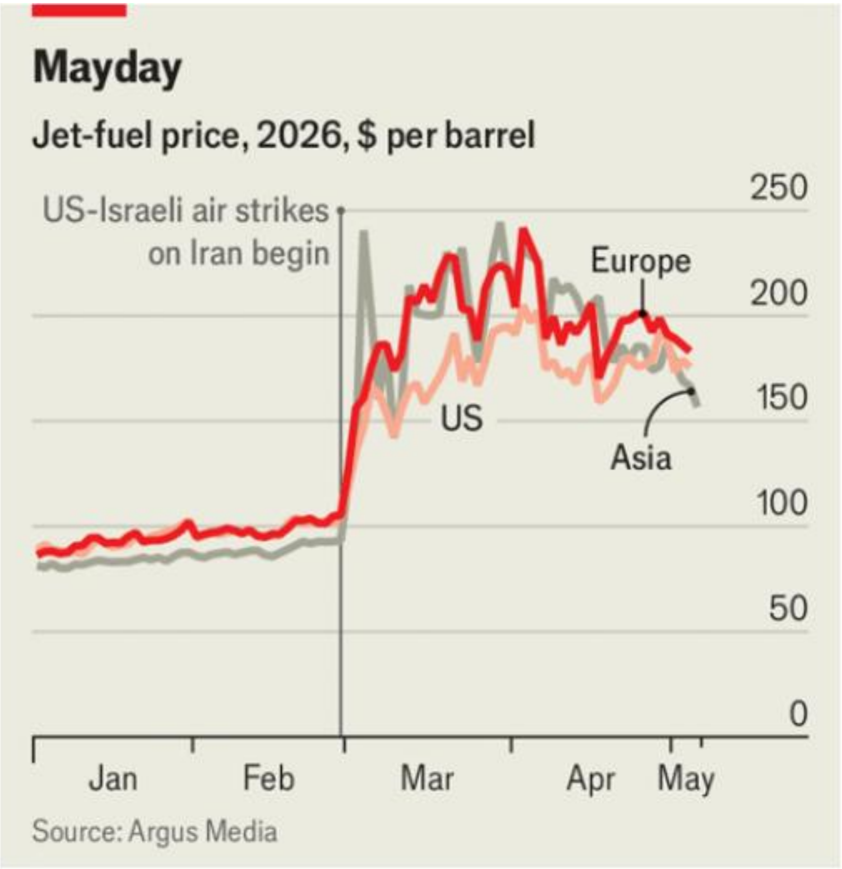
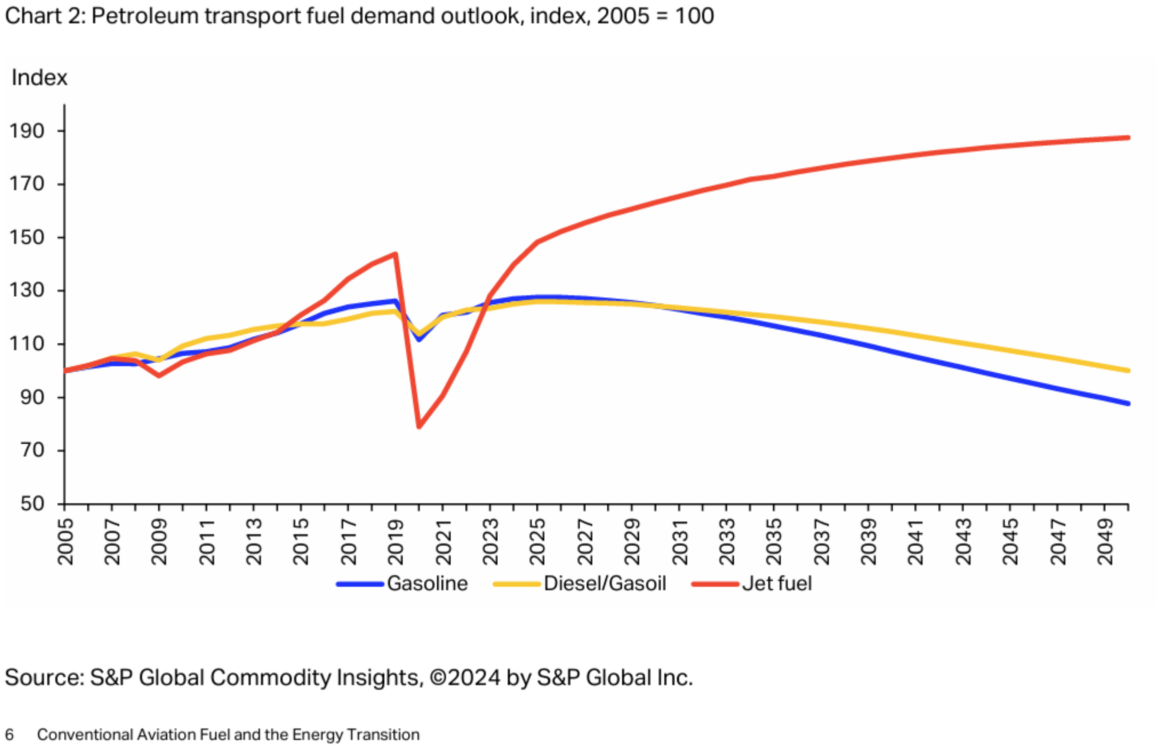


CHART: THE ECONOMIST



Same Tools, Different Thinking?

The long peace of 1945-202x?

IDEAS

THE AGE OF AMERICAN NAVAL DOMINANCE IS OVER

The United States has ceded the oceans to its enemies. We can no longer take freedom of the seas for granted.

By Jerry Hendrix
Photo-illustrations by Oliver Munday

MARCH 13, 2023

SHARE SAVE

The Atlantic

Insurance not finance…

Marine · 'The market is behaving as it should': Correcting the record on war risk cover in the gulf

'The market is behaving as it should': Correcting the record on war risk cover in the gulf

Brokers, P&I executives and the Lloyd's Market Association have spent weeks pushing back against the narrative that marine insurers withdrew from the Strait of Hormuz

Insurance BUSINESS

Thinking About Oil Oligopolies

- Let's do some math!
- Let's talk about Puiseux Series + Oil Oligopolies!!
- Work with Dr. Junhe Chen (RBC)
- and Prof Rob Corless (gentleman of mathematical leisure)

Economic Motivation of Differential-game Approach

- Energy markets involve a number of participants, each with a substantial market share.
- Prices come from supply and demand. Not all producers have the same cost structure. Producers may enter or exit the market depending on prices.
- Differential games allow optimal production strategies to be constructed where prices are endogeneously determined by aggregate outputs.

Literature studying of Differential Games in Energy Finance

- Harris, Howison, Sircar (2010): **Cournot game** and its approximate solution with finite reserve.
  Ledvina, Sircar (2011): **Bertrand games** and its approximate solution.
  Sircar, Ledvina (2012): Oligopoly games with asymmetric costs.
  Chan, Sircar (2015), (2017): **Mean-field games** in both Bertrand game and Cournot game.
  Brown, Funk and Sircar (2017): Monopolistic game with price level Y_t.

Objective Functions for Players

- The player 0 has finite initial reserve x, called "finite-reserve producer". The objective function is:

$$v(x) = \sup_{q_0 \geq 0} \int_0^T \mathrm{e}^{-rt} q_0(t) \left(1 - q_0(t) - \sum_{i=1}^{N-1} q_i^*(t) - s_0 \right) \mathrm{d}t \qquad (2)$$

$$\mathrm{d}x(t) = -q_0(t) \mathbb{1}_{\{x(t)>0\}} \, \mathrm{d}t; \quad X(0) = x.$$

- Player $1, 2, \dots, N-1$ have infinite initial reserve, called "infinite-reserve producer". The objective function for player n is:

$$v_n(x) = \sup_{q_n \geq 0} \int_0^\infty \mathrm{e}^{-rt} q_n(t) \left(1 - q_n(t) - \sum_{i=0, i \neq n}^{N-1} q_n^*(t) - s_n \right) \mathrm{d}t \quad (3)$$

Improvement of the Model

- The traditional model assumes the profit level is constant. So we can add a stochastic term $Y(t)$ for the profit term.
  For reasons of analytic tractability, we choose the model

$$\pi(q, Y(t); s) = Y(t)(1 - s) - q \tag{4}$$

  where the profit level $Y(t)$ has a GBM growth:

$$\begin{aligned} \mathrm{d}Y(t) &= \mu Y(t)\,\mathrm{d}t + \sigma Y(t)\,\mathrm{d}W(t), \\ Y(0) &= y. \end{aligned} \tag{5}$$

Revised Objective Functions

- The finite-reserve producer has the objective function:

$$v(x,y) = \sup_{q_0 \geq 0} \mathbb{E}\left[\int_0^T e^{-rt} q_0 \left(Y(t)(1-s_0) - q_0(t) - \sum_{i=1}^{N-1} q_i^*(t) \right) dt \right]$$

$$dx(t) = -q_0(y,t) \mathbb{1}_{\{x(t)>0\}}\, dt; \quad X(0) = x$$

$$dY(t) = \mu Y(t)\, dt + \sigma Y(t)\, dW(t); \quad Y(0) = y. \tag{6}$$

- The infinite-reserve producer, player n has the objective function:

$$v_n(x,y) = \sup_{q_n \geq 0} \mathbb{E}\left[\int_0^\infty e^{-rt} q_n \left(Y(t)(1-s_n) - q_n(t) - \sum_{i=0, i\neq n}^{N-1} q_i^*(t) \right) dt \right]. \tag{7}$$

HJB equation

- The HJB equation for finite player is

$$rv = \sup_{q_0} q_0 \left(y(1 - s_0) - q_0 - \sum_{i=1}^{N-1} q_i^* - \frac{\partial v}{\partial x} \right) + \mu y \frac{\partial v}{\partial y} + \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 v}{\partial y^2}. \tag{8}$$

- Assuming n active players, the production rates for them are:

$$\begin{aligned} q_0^*(x, y) &= \frac{y(1 + \sum_{i=1}^{n-1} s_i - n s_0) - n \frac{\partial v}{\partial x}}{n + 1} \\ q_k^*(x, y) &= \frac{y(1 + s_0 + \sum_{i=1, i \neq k}^{n-1} s_i - n s_k) + \frac{\partial v}{\partial x}}{n + 1}, \quad 1 \leq k \leq n - 1 \end{aligned} \tag{9}$$

Boundary Value

- So the HJB PDE becomes

$$rv = \frac{n^2}{(n+1)^2}\left(ya_n - \frac{\partial v}{\partial x}\right)^2 + \mu y \frac{\partial v}{\partial y} + \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 v}{\partial y^2}. \tag{10}$$

- When $x = 0$ or $x = \infty$, all active players in the game hold infinite reserve.
  We can derive boundary values in infinite-reserve case (you can ask for details afterwards):

$$v(0,y) = 0, \quad \frac{\partial v}{\partial x}(0,y) = ya_N$$
$$v(\infty,y) = \frac{y^2}{r - 2\mu - \sigma^2}\left(\frac{1 + \sum_{i=0}^{K-1} s_i - Ks_0}{K+1}\right)^2 \tag{11}$$

From PDE to ODE

- We can use the similarity method to transform the PDE to ODE by letting $\xi := \frac{x}{y}$

$$(r-2\mu-\sigma^2)H = \frac{n^2}{(n+1)^2}(a_n - H')^2 - \left(\mu+\sigma^2\right)\xi H' + \frac{1}{2}\sigma^2\xi^2 H''. \quad (12)$$

where $\delta_{n+1} < H'(\xi) \le \delta_n$, $\delta_n = (n+1)s_n - (1 + s_0 + \sum_{i=1}^{n-1} s_i)$, and $n = N-1, N-2, \ldots, K$. And $\xi_b^n = (H')^{-1}(\delta_n)$ is called a "blockading point".

- The boundary value functions are

$$H(0) = 0, \quad H'(0) = a_N.$$

$$H(\infty) = \frac{1}{r-2\mu-\sigma^2}\left(\frac{1+\sum_{i=0}^{K-1} s_i - K s_0}{K+1}\right)^2 \quad (13)$$

Difficulties of the ODE

- The second derivative $H''(0)$ may be infinite, i.e., $\xi = 0$ is a singular point.
  So we cannot apply the naive numerical solution using initial value or boundary value.
- Instead, we can apply the method of dominant balanced to obtain an approximate solution on the interval $[0, \xi_0]$.
- Then use the boundary values $H(\xi_0)$ with terminal value $H(\infty)$ and solve the ODE.

Example

- Assume that $H(\xi) = a_N\xi + P_{\frac{3}{2}}\xi^{\frac{3}{2}}$ and obtain

$$\left(a_N(\mu - r) + \frac{9N^2{P_{\frac{3}{2}}}^2}{4\,(N+1)^2}\right)\xi + \left(-\frac{1}{8}\sigma^2 P_{\frac{3}{2}} + 1/2\,P_{\frac{3}{2}}\mu - P_{\frac{3}{2}}r\right)\xi^{\frac{3}{2}} = 0. \tag{15}$$

- The method of dominant balance method ignores coefficient of the smaller-order term $\xi^{\frac{3}{2}}$.

  Setting the term of $a_N(\mu - r) + \frac{9N^2{P_{\frac{3}{2}}}^2}{4(N+1)^2} = 0$ gives the first term of Puiseux series

$$P_{\frac{3}{2}} = -\frac{2\sqrt{a_N\,(r-\mu)}\,(N+1)}{3N}. \tag{16}$$

- Then we continue in the case of $H(\xi) = a_N\xi + P_{\frac{3}{2}}\xi^{\frac{3}{2}} + P_2\xi^2$ to obtain P_2.

Puiseux Series

- The first 4 terms of Puiseux series are

$$
\begin{aligned}
P_{\frac{3}{2}} &= -\frac{2\sqrt{a_N(r-\mu)}\,(N+1)}{3N}\\
P_2 &= \frac{(N+1)^2(\sigma^2-4\mu+8r)}{48N^2}\\
P_{\frac{5}{2}} &= \frac{(N+1)^3(4\mu-8r-\sigma^2)(4r+4\mu-\sigma^2)}{2880\sqrt{a_N(r-\mu)}}\\
P_3 &= -\frac{(N+1)^4(4r+4\mu-\sigma^2)(16\mu-7\sigma^2-8r)(4\mu-8r-\sigma^2)}{207360N^4a_N(r-\mu)}\\
&\cdots\cdots
\end{aligned}
\tag{17}
$$

Numerical Solution

- Once we obtain an approximate solution on interval $[0, \xi_0]$, we can use the finite difference method

$$\frac{n_i^2}{(n_i+1)^2}\left(a_{n_i} - \frac{H_{i+1}-H_{i-1}}{2h}\right)^2 - \left(\mu+\sigma^2\right)\xi_i\frac{H_{i+1}-H_{i-1}}{2h} + \frac{1}{2}\sigma^2\xi_i^2\frac{H_{i+1}+H_i-H_{i-1}}{h^2} - (r-2\mu-\sigma^2)H_i = 0. \tag{18}$$

where n_i is determined by $\delta_{n_i-1} \leq \frac{H_{i+1}-Hi-1}{2} < \delta_{n_i}$ and $i = 1, 2, \ldots, M-1$.
The initial value is $H(\xi_0)$.
The terminal value is $H(\xi_M) = \frac{1}{r-2\mu-\sigma^2}\left(\frac{1+\sum_{i=0}^{K-1} s_i - Ks_0}{K+1}\right)^2$ by letting ξ_M is a sufficiently large value.

Plot of the Solution

Assume that $s_0 = 0.05, s_1 = 0.30, s_2 = 0.32, r = 0.05, \mu = 0.01, \sigma = 0.07$. The plot below shows the solution $H(\xi)$, where the green vertical lines are "blockading points":

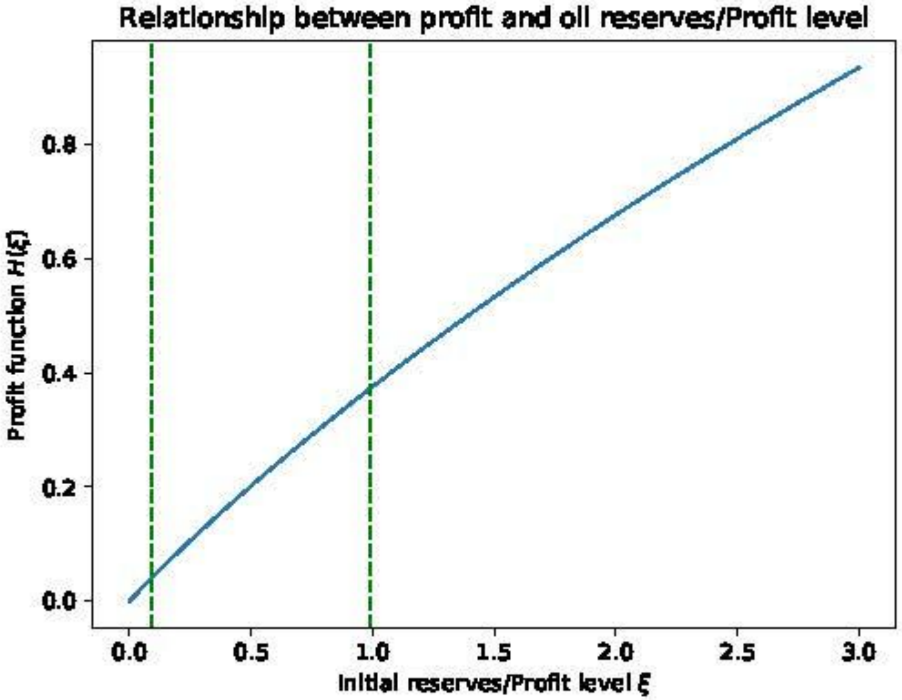


Figure: Plot of profit function

Plot of the Solution

Reverting to $H(x, y) = y^2 v(\frac{x}{y})$:

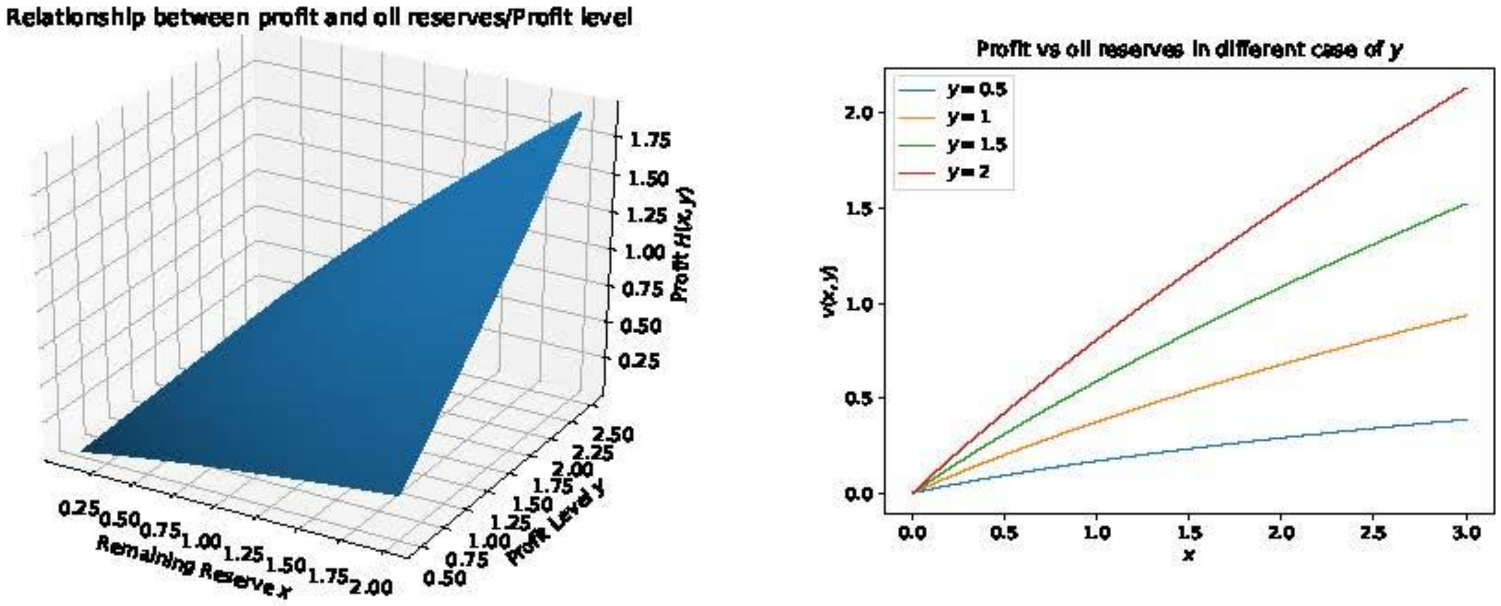


Figure: Plot of profit function

Plot of Production Rate

The plot of the finite player's production rate is

$$q_0^*(x,y) = \frac{ny}{n+1}\left(a_n - H'\left(\frac{x}{y}\right)\right). \tag{19}$$

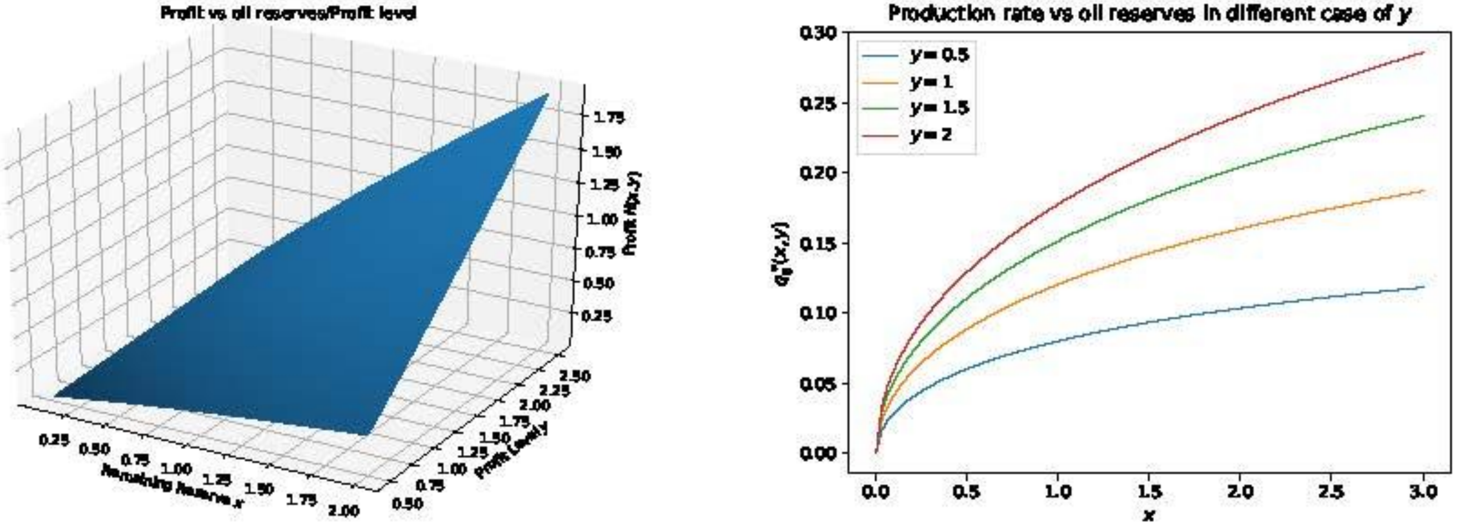


Figure: Plot of Production Rate

Conclusions

- Do economic drivers still dominate decision making around real energy assets?
- What if a country needs to produce oil to prove what teams it's on?
  - So, the country produces it at an economically suboptimal level?
  - The math just got a lot harder
- Is "Risk is good" (finance) world transitioning to "Risk is bad" (insurance) world?
- Markets thinking still has a place
- The deck may be shuffling again as it did after 2008: new opportunities!