Algorithmic Treatment of Filippov-Type Switched and Sliding-Mode ODE and DAE Systems with IFDIFF [1]

Ordinary differential equations (ODEs) and differential-algebraic equations (DAEs) with implicit (state-dependent) switches and/or state jumps-common in applications from mechanics and disease modelling-challenge general-purpose integrators, often producing incorrect solutions without being noticed. Proper application of Filippov theory [2], on the other hand, requires extensive manual effort to derive switching planes, model convex combinations, and manifold dynamics. Furthermore, even minor model changes often require substantial rework.

This talk recaps key aspects of Filippov theory before presenting the IFDIFF Toolbox for MATLAB, which automates this entirely: Given model code with non-differentiable operators (min, max, abs, sign, if-branching), IFDIFF generates required switching functions on-the-fly during integration and detects switching points and sliding manifold entry/exit up to machine precision. No manual model modifications are required.

As integrators, unmodified solvers from the Matlab ODE solvers suite like ode45 (embedded Runge-Kutta 4(5) for non-stiff ODE) or ode15s (multi-step BDF variant for stiff ODE and DAE) can be used, and the generated solution objects integrate seamlessly with existing code.

First order forward sensitivities can be generated using variational differential equations or finite-differencing, with appropriate sensitivity updates at the switching points. The automatically generated switching functions are continuously differentiable and ready to be re-used in other contexts.

We demonstrate IFDIFF's capabilities on selected prototypical examples from mechanics and biology, showing how accurate simulations and model sensitivities are obtained.

[1] https://andreassommer.github.io/ifdiff/
[2] A. F. Filippov: "Differential Equations with Discontinuous Righthand Sides" (in Russian), Mat. Sb. (N.S.), 1960, Vol 93(1)