Preconditioning and Linearly Implicit Time Integration for the Serre-Green-Naghdi Equations

The Serre-Green-Naghdi (SGN) equations are fluid models that incorporate dispersion (through higher order asymptotic expansions) into the traditional shallow water equation (SWE). While the presence of dispersion results in the modeling of important physical effects, the SGN equations contain a differential PDE constraint which poses a key numerical challenge.

This talk will focus on developing time integration and preconditioning strategies for handling the differential PDE constraint. First, we establish a constant coefficient preconditioner and rigorous bounds on the preconditioned conditioning number. The conditioning bounds incorporate the effects of bathymetry in two dimensions, are quasi-optimal within a class of constant coefficient operators, highlight fundamental scalings for a loss of conditioning, and ensure mesh independent performance for iterative Krylov methods.

Utilizing the conditioning bounds, we devise and test two time integration strategies for solving the full SGN equations. The first class combines classical explicit time integration schemes (4th order Runge-Kutta and 2nd–4th order Adams-Bashforth) with the new preconditioner. The second is a linearly implicit scheme where the differential constraint is split into a constant coefficient implicit part and remaining (stiff) explicit part. The linearly implicit methods require a single linear solve of a constant coefficient operator at each time step. We provide a host of computational experiments that validate the robustness of the preconditioners, as well as full solutions of the SGN equations including solitary waves traveling over an underwater shelf (in 1d) and a circular bump (in 2d).