A fourth order stencil for parabolic PDEs under Dirichlet boundary conditions

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ABSTRACT

We present a simple fourth order space semi-discretization, based on finite differences, for parabolic problems in rectangular domains under Dirichlet boundary conditions and an initial condition. The advantage of the new approach is that it substantially reduces the number of grid-points used when compared with the standard second order approximation to get a similar accuracy, this implies a great reduction of the stiffness in the resulting ODEs, consequently these ODEs can even be integrated with some adequate explicit methods. We show that these stencils easily adapt to multidimensional parabolic problems on hyper-rectangles. We illustrate the convergence orders with several examples and provide the ideas for the proofs of the convergence results.

References

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