Sub-ODEs Simplify Taylor Series Algorithms for Ordinary Differential Equations

A Taylor method for solving an ordinary differential equation (ODE) initial-value problem computes a truncated Taylor series (TS) expansion of the solution at a given point and advances the solution by summing the TS with an appropriate step size. To find Taylor coefficients for the solution, automatic differentiation is typically used, which requires implementing the four basic arithmetic operations (BAOs) and standard functions (such as exp, sin) on Taylor polynomials.

Many commonly used functions satisfy simple ODEs that involve only BAOs; for example, exp(u(t)) satisfies v' = v * u'. We introduce a novel sub-ODE method that directly incorporates the ODEs of such functions into the evaluation process. This approach streamlines implementation: each function that satisfies a simple ODE is added to the "Taylor library" with a few lines of code, eliminating the need for separate recurrence formulas.

However, embedding sub-ODEs typically transforms the original ODE into a differential-algebraic system. We present results showing that this approach is theoretically sound, regardless of the number of sub-ODEs incorporated.

Finally, we introduce our sub-ODE-based MATLAB ODE solver and show that its performance compares favorably with existing solvers from the MATLAB ODE suite.

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