A Piecewise Deterministic Approach for Parameter Estimation in Multi-Dimensional Stochastic Differential Equation Models

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Many processes, especially in biology, follow certain basic principles but are influenced by intrinsic stochasticity that may significantly alter the system's behaviour both quantitatively and qualitatively. These processes can often be adequately modelled by a system of nonlinear stochastic differential equations (SDE)

$$dX_t = f(t, X_t, p)\, dt + D\, dW_t \qquad X_0 = x_0(p)$$

with state vector $X_t \in \mathbb{R}^{n_x}$, whose every component might be affected by a diffusion of constant intensity, i.e. D is an $n_x \times n_x$ diagonal matrix, and dW_t denotes an n_x-dimensional white noise process. Both, the *drift function* f and the initial state X_0 may depend on parameters $p \in \mathbb{R}^{n_p}$ that are to be estimated from measurements.

On short time scales, such SDE resemble ordinary differential equations (ODE) processes, if diffusion is not the dominating part of the system. From this observation, the idea rises to split the time horizon into multiple intervals, in which only the drift part of the SDE, i.e. the associated ODE, is used for simulating the process. The *jumps* (discontinuities) between the intervals may be interpreted as cumulated stochasticity of the preceding interval.

Simply concatenating the interval ODE solutions would mean a complete decoupling of the interval processes and, in general, leads to wrong conclusions. Instead, the jumps can used for regularization in the parameter estimation problem.

The proposed piecewise deterministic approach allows using state-of-the-art ODE solvers and efficient derivative-based optimization methods. It can handle multi-dimensional systems, equality and inequality constraints on the states and parameters, and poses no structural demands on the measurement data (arbitrary measurement times, direct or indirect, full or partial, exact or noisy). The method can also be used for hidden state or trajectory reconstruction (see figure 1).

A convergence result and applications to problems in systems biology and finance will be presented.

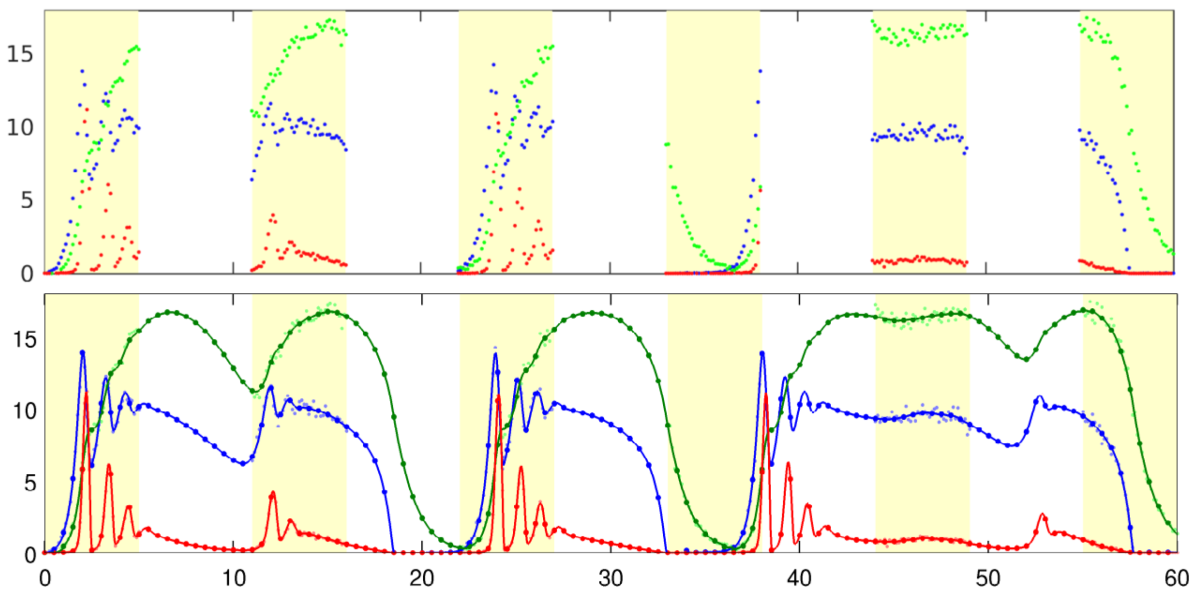

Figure 1: Calcium oscillator model (a.u., Kummer et al, 2005). The observed oscillations are due to intrinsic stochasticity. Top: Intermittent observations with measurement errors. Bottom: Reconstructed trajectory. Enlarged dots mark the beginning of a new approximation interval. The "bump" in the last unobserved time range is a successful reconstruction of the original trajectory.