Title: Can numerical integrators be contractive on Riemannian manifolds?

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Abstract.
Recently, Sherry et al. (2023) reconsidered the pioneering work of Dahlquist and Jeltsch (1979) on circle-contractivity for the study of neural networks. This theory works in Euclidean spaces, but the importance of neural networks set on Riemannian manifolds seems to be increasing and there is a need to develop the theory of contractive numerical methods also in such a setting.
We present some ideas from Arnold et al. (2024) where some simple numerical methods for Riemannian manifolds are studied. We consider whether these methods can be non-expansive when applied to non-expansive vector fields. For the geodesic implicit Euler method, which also feature in the proximal gradient method for optimisation, we find that its behaviour is strongly dependent on the sectional curvature of the manifold. As opposed to the Euclidean case, we now also have to be careful about whether the nonlinear equations to be solved in each time step has a unique solution or not.

References

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