

Unconditionally Positive and Conservative Linear Multistep Methods for Production-Destruction Systems

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A wide variety of mathematical models for real life problems are given in the form of positive and conservative Production-Destruction differential Systems (PDS) [1]. Patankar-type schemes are linearly implicit integrators for PDS, traditionally based on Runge-Kutta schemes and specifically designed to be unconditionally conservative and positive [2,3]. Here we extend the Patankar approach to linear multistep methods and prove that the resulting schemes retain, with no conditions on the step size, the positivity of the solution and the linear invariant of the continuous-time system. Moreover, we provide results on arbitrarily high order of convergence achieved through an embedding technique for the Patankar weights denominators. Finally, we report numerical tests that confirm the theoretical results and show that Modified Patankar Linear Multistep Methods are competitive and can have higher accuracy and better performance than Modified Patankar Runge-Kutta methods.

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References

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