

Dense Output with Quasi-Interpolants for Boundary Value Problems

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In this lecture, we will discuss the use of quasi-interpolants for computing dense output in boundary value problems. We will provide a brief review of the techniques used in available codes and then present two specific quasi-interpolation algorithms.

First, we will discuss the quasi-interpolation schemes implemented in the C++ library QIBSH++, which are derived from the BS linear multistep methods. Next, we will explore the Hermite-Birkhoff quasi-interpolation schemes, which are derived from the one-step class of Hermite-Obreshkov methods. To calculate the required Lie derivatives, we will employ Infinity Computer Arithmetics. We will also provide a brief description of a preliminary Matlab class that implements a subset of the Infinity Computer Arithmetic for this purpose.

We will show the practical application of these two quasi-interpolants in computing high-order dense output for super-convergent collocation Runge-Kutta methods, such as the Gauss-Lobatto and Gauss-Legendre methods. Additionally, we will share some preliminary results obtained from their implementation in the bvptwp code.

References

- [1] M.F., Sestini A., The BS class of Hermite spline quasi-interpolants on nonuniform knot distributions, *BIT Numerical Mathematics* 49 (3), 611 – 628 (2009).
- [2] Bertolazzi E.; Falini, A, Falini A.; M. F., The Object Oriented C++ library QIBSH++ for Hermite spline Quasi Interpolation, arXiv (2022), DOI: 10.48550/arXiv.2208.03260
- [3] Iavernaro F., M. F., Mukhametzhanov M.S., Sergeyev Y.D., Computation of higher order Lie derivatives on the Infinity Computer, (2021) *Journal of Computational and Applied Mathematics* 383, art. no. 113135,
- [4] Falini A., M. F. and Sestini A., Hermite–Birkhoff spline Quasi–Interpolation with application as dense output for Gauss–Legendre and Gauss–Lobatto Runge–Kutta schemes, (2023) submitted.