Optimal control of an SIR epidemic through contact reduction

The early phase of the Covid-19 pandemic raised the question of how best to mitigate the effects of an epidemic purely by reducing the amount of contact between individuals. We consider this as an optimal control problem, in which the contact rate is the control, and in the context of the very simplest epidemiological model, known as the SIR model. We assume that the control can be applied only up to a finite future time, but consider the longer-term outcome of the epidemic in the objective function. If one assumes that there is no cost to applying the control, the solution becomes particularly simple (though practically unrealistic). After studying this solution, we also consider more realistic situations that include a cost of social distancing and a penalty term designed to avoid hospital overcrowding. We consider the formulation of this problem using both the Pontryagin and Hamilton-Jacobi-Bellman approaches, and present numerical solutions using both approaches that are in agreement.